A Coupled-Mode Theory Approach for Consolidating Nonlinearities with Quasinormal Modes

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I Introduction

- **Objective** → Develop a **general** and **rigorous nonlinear framework** capable of correctly **handling non-Hermitian** systems that support **quasi-normal modes (QNMs)** with arbitrarily low quality factors.
- **Motivation** → Aid the trend of **shrinking resonant cavity size** with theoretical **tools** that are capable to correctly capture their physical nature and allow for **efficient design**.
- **Applications** → The presented framework can model both classical **guided-wave** systems (e.g. a traveling-wave ring resonator) or contemporary **free-space** systems (e.g. a highly dispersive plasmonic core-shell).

II Nonlinear framework development

Perturbation Theory

- The framework is developed under **classical electromagnetism**, namely the **Maxwell equations** and **Lorentz reciprocity** theorem [1].

$$\nabla \times \mathbf{E}_0 = -j\tilde{\omega}_0\mu_0\mathbf{H}_0$$
$$\nabla \times \mathbf{H}_0 = j\tilde{\omega}_0\varepsilon_0\tilde{\varepsilon}_r(\tilde{\omega}_0)\mathbf{E}_0 \quad (+j\tilde{\omega}\mathbf{P}_{\mathrm{pert}})$$

- The general **unconjugated** Lorentz reciprocity theorem is applied which allows for materials with arbitrarily high **ohmic losses** and resonators with potentially important **light leakage** [2]; PMLs are also utilized in the formulation to halt the divergence and make the profile square integrable (regularize).

$$\frac{\Delta\tilde{\omega}}{\tilde{\omega}_0} = -\frac{\iiint_{V_p} \mathbf{P}_{\mathrm{pert}}\cdot\mathbf{E}_0\,\mathrm{d}V}{\iiint_{V+V_{\mathrm{PML}}} \varepsilon_0\frac{\partial\{\omega\tilde{\varepsilon}_r(\omega)\}}{\partial\omega}\mathbf{E}_0\cdot\mathbf{E}_0\,\mathrm{d}V - \iiint_{V+V_{\mathrm{PML}}}\mu_0\mathbf{H}_0\cdot\mathbf{H}_0\,\mathrm{d}V}$$

- Resonant frequency shift is simply calculated by the **projection** of the **normalized perturbation** on the **normalized QNM** profile [2,3].

$$\mathbf{P}_{\mathrm{pert},n} = \mathbf{P}_{\mathrm{pert}}/\boxed{\sqrt{\mathcal{Q}_{\mathrm{QNM}}}} \qquad \mathbf{E}_n = \mathbf{E}_0/\boxed{\sqrt{\mathcal{Q}_{\mathrm{QNM}}}}$$

$$\mathcal{Q}_{\mathrm{QNM}} = \iiint_{V+V_{\mathrm{PML}}} \varepsilon_0\frac{\partial\{\omega\tilde{\varepsilon}_r(\omega)\}}{\partial\omega}\mathbf{E}_0\cdot\mathbf{E}_0\,\mathrm{d}V - \iiint_{V+V_{\mathrm{PML}}}\mu_0\mathbf{H}_0\cdot\mathbf{H}_0\,\mathrm{d}V$$

Kerr nonlinearities and consolidation with CMT

- **Kerr nonlinearity** is chosen as an indicative example

$$\mathbf{P}_{\mathrm{pert}} = \frac{1}{3}\varepsilon_0^2 c_0 n_2 \mathrm{Re}\{\tilde{\varepsilon}_r\}\left[2(\mathbf{E}_0\cdot\mathbf{E}_0^*)\mathbf{E}_0 + (\mathbf{E}_0\cdot\mathbf{E}_0)\mathbf{E}_0^*\right]$$

- **Stored energy** should be introduced to transform $\Delta\tilde{\omega}$ in a form that is appropriate for CMT; nevertheless it **diverges**. **Resistive quality factor** is used to implicitly define the stored energy and **lift** the dependence on the computational domain size.

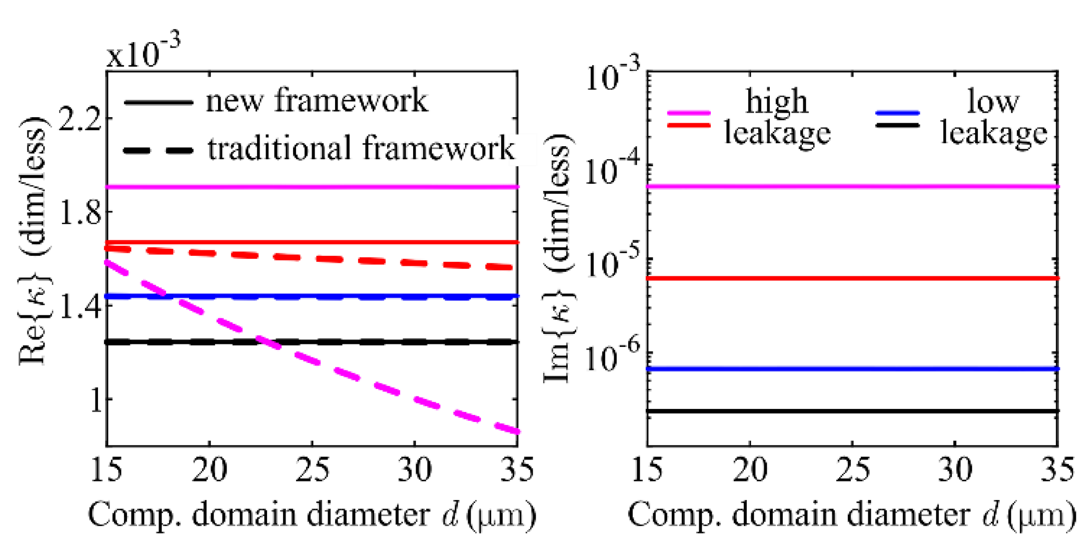
$$\frac{\Delta\tilde{\omega}}{\tilde{\omega}_0} = -\frac{\varepsilon_0^2 c_0\iiint_{V_p} n_2\mathrm{Re}\{\tilde{\varepsilon}_r\}|\mathbf{E}_0|^2(\mathbf{E}_0\cdot\mathbf{E}_0)\,\mathrm{d}V}{\iiint_V \varepsilon_0\frac{\partial\{\omega\tilde{\varepsilon}_r(\omega)\}}{\partial\omega}\mathbf{E}_0\cdot\mathbf{E}_0\,\mathrm{d}V - \iiint_V\mu_0\mathbf{H}_0\cdot\mathbf{H}_0\,\mathrm{d}V}\boxed{\frac{|a|^2}{\frac{Q_{\mathrm{res}}P_{\mathrm{res}}}{\omega_0}}}$$

- The strength of the nonlinearity is described by the ***nonlinear feedback parameter***.

$$\tilde{\kappa} = \left(\frac{c_0}{\omega_0}\right)^3\frac{\iiint_{V_p} n_2\mathrm{Re}\{\tilde{\varepsilon}_r(\omega_0)\}|\mathbf{E}_0|^2(\mathbf{E}_0\cdot\mathbf{E}_0)\,\mathrm{d}V}{\frac{1}{\varepsilon_0^2}\left[\iiint_V \varepsilon_0\frac{\partial\{\omega\tilde{\varepsilon}_r(\omega)\}}{\partial\omega}\mathbf{E}_0\cdot\mathbf{E}_0\,\mathrm{d}V - \iiint_V\mu_0\mathbf{H}_0\cdot\mathbf{H}_0\,\mathrm{d}V\right]n_2^{\max}}\frac{\omega_0}{4Q_{\mathrm{res}}P_{\mathrm{res}}}$$

✓ $\tilde{\kappa}$ is complex –thus, **phase** effects can **affect** the **linewidth** of a resonance– and **independent** of the **computational domain size**.

× In classical approaches $\tilde{\kappa}$ is real and depends on the integration domain size (spatial divergence of the stored energy).



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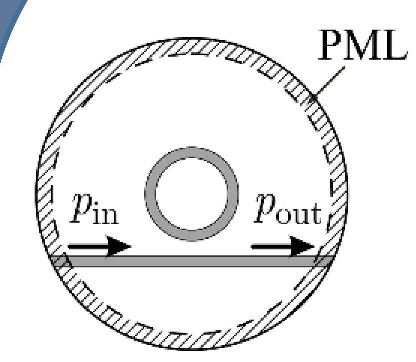
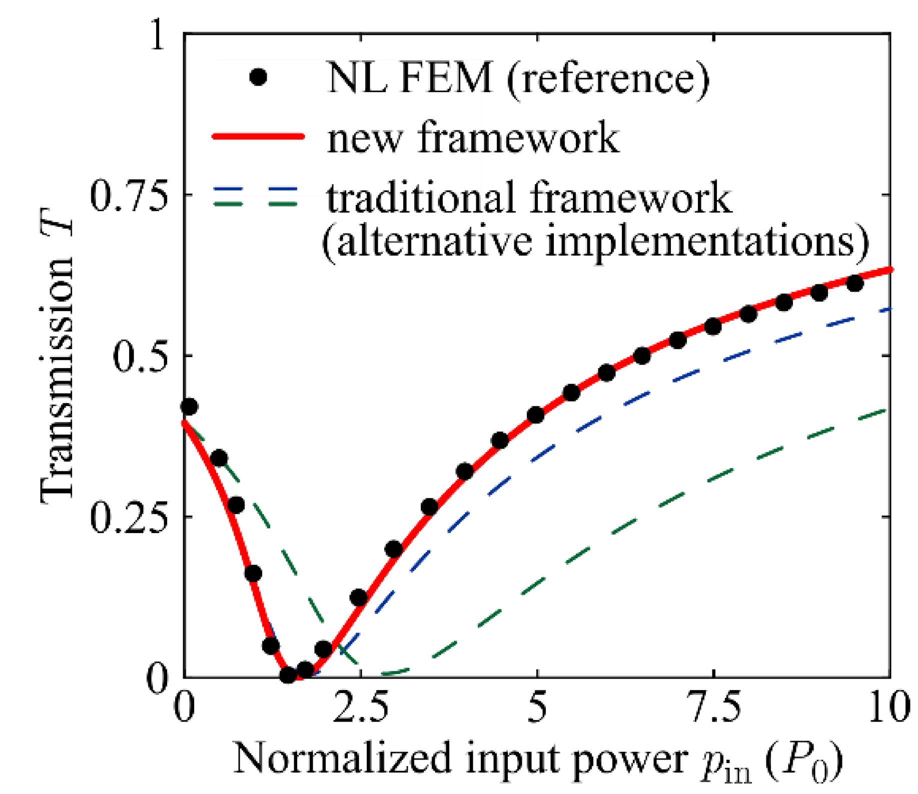
III Nonlinear framework applications

Guided-wave system (slab ring resonator)

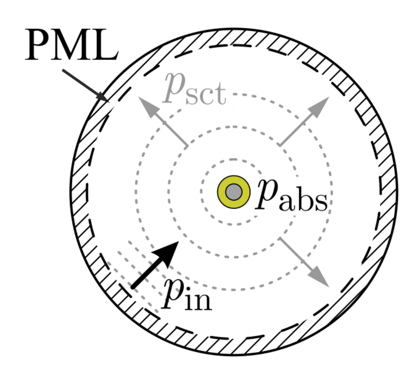


- Typical **slab ring resonator** with R = 0.79 μm, w = 200 nm, and g = 200 nm to fulfill the critical coupling condition with the bus waveguide.
- The framework leads to a system of simple **polynomial equations** which **correctly predicts** the nonlinear transmission, even for **highly leaky resonances** ($Q_{\mathrm{rad}} \sim 140$) where similar **traditional approaches** notably **fail**.

$$\frac{p_{\mathrm{out}}}{p_{\mathrm{in}}} = \frac{(\delta+p_i)^2 + (1-r_Q+r_{\tilde{\gamma}}p_i)^2}{(\delta+p_i)^2 + (1+r_Q+r_{\tilde{\gamma}}p_i)^2}$$
$$p_{\mathrm{in}} - p_{\mathrm{out}} - p_i = r_{\tilde{\gamma}}p_i^2$$

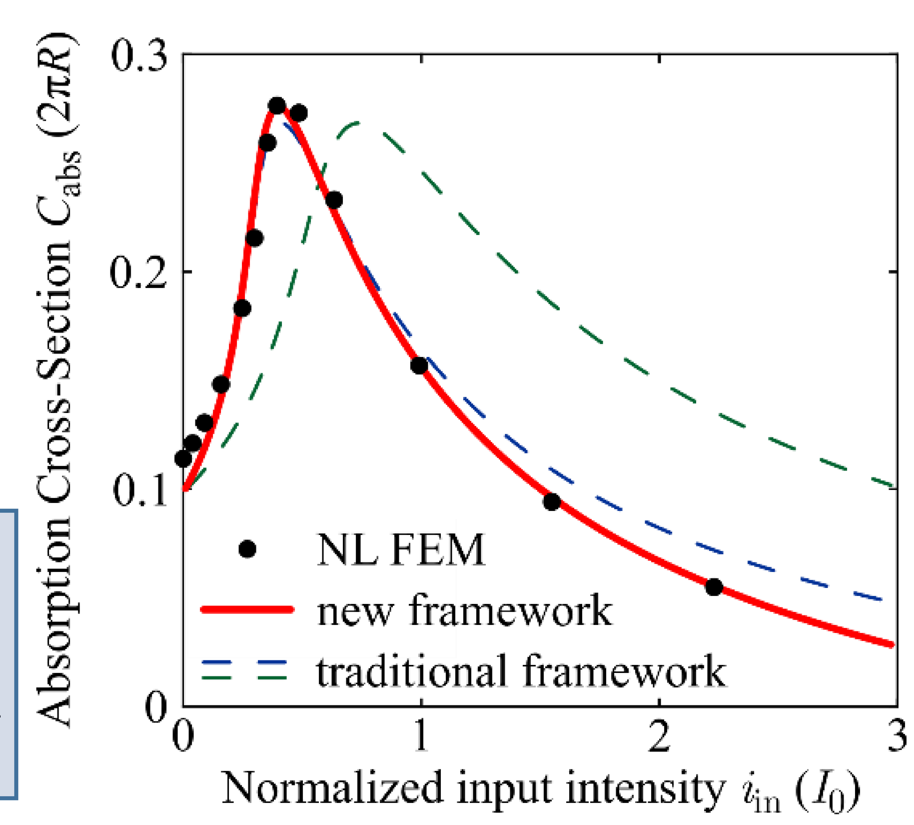


Free-space system (plasmonic core-shell)



- Highly dispersive **plasmonic core-shell** with a nonlinear dielectric core, with R = 152 nm and w = 24 nm. Its quality factor is very low and equals $Q_{\mathrm{rad}} \sim 40$.
- The system of CMT equations is similar but we introduce an extra **dependence** of the **radiation** and **absorption quality factors** on the **input power**, driven by the different field profile due to the nonlinearity.

$$C_{\mathrm{abs}} = (m+1)\frac{\lambda_0}{2\pi}\times$$
$$\times\frac{2r_{\mathrm{abs}}(p_{\mathrm{in}})r_{\mathrm{rad}}(p_{\mathrm{in}})}{(\delta+p_{\mathrm{rad}})^2 + (1+r_Q+r_{\tilde{\gamma}}p_{\mathrm{rad}})^2}$$



IV Future expansions

- Extend to **multi-mode framework**. Useful especially in **low-quality-factor systems** where modes might spectrally overlap.
- Multi-mode QNM frameworks have recently appeared in the literature, but for linear systems [3,4,5].
- Immediate next step: implement a self-induced **nonlinear phenomenon** (e.g. the Kerr effect) in such a **general multimode framework**.

$$C_{\mathrm{abs}}(I_{\mathrm{in}}) \propto \int_{V_p}\sum_k \alpha_k(I_{\mathrm{in}})\mathbf{E}_{k,\mathrm{QNM}}\cdot\mathbf{P}_{\mathrm{pert}}\mathrm{d}V$$

V Conclusions

A **rigorous** perturbation theory framework for studying nonlinear material modifications in **leaky optical cavities** is developed and numerically validated. Both **guided** and **free-space** systems are examined, paving the way for the application of this nonlinear perturbation theory approach to a broad range of nonlinearity types in leaky resonant systems.

VI References

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